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1. R. J. Sasiela, "Strehl ratios with various types of anisoplanatism," J. Opt. Soc. Am. A **9,** 1398–1405 (1992).

2. W. Zhao and E. Bourkoff, "Generation, propagation, and amplification of dark solitons," J. Opt. Soc. Am. B **9,** 1134–1144 (1992).

3. J. P. Pratt and V. P. Heuring, "Designing digital optical computing systems: power distribution and cross talk," Appl. Opt. **31,** 4657–4661 (1992).

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Strehl ratios with various types of anisoplanatism

Richard J. Sasiela

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Abstract

There are many ways in which the paths of two waves through turbulence can become separated, thereby leading to anisoplanatic effects. Among these are a parallel path separation, an angular separation, one caused by a time delay, and one that is due to differential refraction at two wavelengths. All these effects can be treated in the same manner. Gegenbauer polynomials are used to obtain an approximation for the Strehl ratio for these anisoplanatic effects, yielding a greater range of applicability than the Maréchal approximation.

1. INTRODUCTION

Adaptive-optics systems are used to correct images of objects. These systems work by measuring the phase distortion on a downpropagating wave called a beacon and applying the negative of that phase to a deformable mirror. If this is done well, then the image of the beacon is close to diffraction limited; and if a laser beam is projected along the corrected path, it will have propagation characteristics approaching those of a wave propagating in vacuum. It is not possible to make a perfect correction; one of the major error sources is due to the fact that the rays of the object to be imaged or the laser beam to be propagated are along a path displaced from that of the beacon. A measurement of this degradation is the Strehl ratio, which is the ratio of the intensity of the actual beam on axis to that of a diffraction-limited beam.

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\documentstyle[osa,manuscript]{revtex}  % DON'T CHANGE %
\newcommand{\MF}{{\large{\manual META}\-{\manual FONT}}}
\newcommand{\manual}{rm}        % Substitute rm (Roman) font.
\newcommand\bs{\char '134 }     % add backslash char to \tt font %
%
\begin{document}                % INITIALIZE - DONT CHANGE % %  %

\title{Strehl ratios with various types of anisoplanatism}

\author{Richard J. Sasiela }

\address{Lincoln Laboratory, Massachusetts Institute of Technology,
Lexington,  Massachusetts 02173-9108} %

\maketitle
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\section{ INTRODUCTION}
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This displacement can have several causes. The receiving and the transmitting apertures may be displaced from each other owing to misalignment or vignetting of the beams. The paths can be separated in angle, for instance, when the object to be imaged is different from the beacon. The correction is applied with a time delay after the measurements. In this time the turbulence is displaced by winds and slewing of the telescope. The paths may be separated because the beacon and the imaging wavelengths differ, in which case refraction operates differently on the two waves. All the effects are typically present simultaneously.

These anisoplanatisms have been treated separately in the past[1–7]; ...

2. STREHL RATIO WITH ANISOPLANATISM

For a perfect correction the paths of the beacon signal and the imaging or projected laser should be the same. In general, this is not possible to achieve, and there is a degradation in performance caused by time delays, displacement of the two paths by translation and angle, and differences in wavelength of the beacon and the measurement or projecting systems.

The effects of displacement, angular mispointing, time delay, and atmospheric dispersion can each be treated as an anisoplanatic effect. In fact, if all the effects are present simultaneously, they can be combined to get a total offset of the measurement from the imaging paths. In this section the effect of a general displacement on the Strehl ratio is determined.

The Strehl ratio (SR) for a circular aperture[7] from the Huygens–Fresnel approximation is

$$\mathrm{SR} = \frac{1}{2\pi} \int \mathrm{d}\boldsymbol{\alpha}\, K(\alpha)\, \exp\left[-\frac{\mathcal{D}\left(\boldsymbol{\alpha}\right)}{2}\right]. \tag{1}$$

The integral is over a circular aperture of unit radius, $\mathcal{D}(\boldsymbol{\alpha})$ is the structure function, and $K(\alpha)$ is a factor times the optical transfer function given by

$$K(\alpha) = \frac{16}{\pi}\left[\cos^{-1}(\alpha) - \alpha\left(1-\alpha^2\right)^{1/2}\right] U(1-\alpha), \tag{2}$$

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\section{ STREHL RATIO WITH ANISOPLANATISM}
\label{SR}
For a perfect correction the paths of the beacon signal and the imaging or projected laser should be the same. In general, this is not possible to achieve, and there is a degradation in performance caused by time delays, displacement of the two paths by translation and angle, and differences in wavelength of the beacon and the measurement or projecting systems.

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The Strehl ratio (SR) for a circular aperture \cite{7} from the Huygens--Fresnel approximation is \begin{eqnarray}{\rm SR} ={1 \over {2\pi }}\int {{\rm d}\bbox \alpha }\,K(\alpha)\,\exp \,\left[{-{{{\cal D}\left({\bbox \alpha } \right)} \over 2}} \right].\end{eqnarray} The integral is over a circular aperture of unit radius, ${\cal D}({\bbox \alpha })$ is the structure function, and $K(\alpha)$ is a factor times the optical transfer function given by \begin{eqnarray}K(\alpha)={{16} \over \pi }\left[{\cos ^{- 1}(\alpha)-\alpha \left({1-\alpha ^2} \right)^{1/ 2}} \right]\,U(1- \alpha),\end{eqnarray} where $U(x)$ is the unit step function defined as

$$U(x) = 1 \quad \text{for} \quad x \geq 0\,,$$
$$U(x) = 0 \quad \text{for} \quad x < 0\,. \tag{3}$$

To find the Strehl ratio, one must first determine the structure function. It was found by Fried[4] for angular anisoplanatism. If the source is collimated and a general displacement is introduced, his expression for a wave propagating from ground to space becomes

$$\mathcal{D}(\alpha D) = 2(2.91)\, {k_0}^2 \int\limits_0^\infty \mathrm{d}z\, {C_n}^2(z) \Big[(\alpha D)^{5/3} + d^{5/3}(z) - \tfrac{1}{2}\,|\boldsymbol{\alpha} D + \boldsymbol{d}(z)\,|^{5/3} - \tfrac{1}{2}\,|\,\boldsymbol{\alpha} D - \boldsymbol{d}(z)\,|^{5/3}\Big]\,, \tag{4}$$

where ${C_n}^2(z)$ is the turbulence strength as a function of altitude; $k_0 = 2\pi/\lambda$, where λ is the wavelength of operation; D is the aperture diameter; and $\boldsymbol{d}(z)$ is the vector displacement of the two paths.

The sums of the terms in brackets almost cancel, thus causing difficulties if one tries to evaluate this integral numerically. The terms in the absolute-value sign are equal to

$$|\,\boldsymbol{\alpha} D \pm \boldsymbol{d}(z)\,|^{5/3} = \Big[(\alpha D)^2 \pm 2\alpha D\, d(z)\cos(\varphi) + d^2(z)\Big]^{5/6}\,, \tag{5}$$

where is the angle between $\boldsymbol{\alpha}$ and $\boldsymbol{d}(z)$. This expression can be simplified and the numerical difficulties can be eliminated by using Gegenbauer polynomials.[8] Their generating function is

$$\left(1 - 2ax + a^2\right)^{-\lambda} = \sum_{p=0}^{\infty} {C_p}^{\lambda}(x)\, a^p. \tag{6}$$

These functions are sometimes referred to as ultraspherical functions because they are a generalization of the Legendre polynomials $P_n(t)$, whose generating function is

$$\left(1 - 2ax + a^2\right)^{-1/2} = \sum_{p=0}^{\infty} P_p(x)\, a^p. \tag{7}$$

The Gegenbauer polynomials with the cosine of a variable as the argument are given in Eq. (8.934 #2) of Ref. 8 and can be rewritten as

$${C_p}^{\lambda}[\cos(\varphi)] = \sum_{m=0}^{p} \frac{\Gamma\,[\lambda + m]\;\Gamma\,[\lambda + p - m]\cos[(p - 2m)\varphi]}{m!\,(p - m)!\;(\Gamma\,[\lambda])^2}, \tag{8}$$

```
where
$U\left(  x \right)$  is the unit step function defined as
\begin{eqnarray} U( x )&=&1\,\,\,\,\,\,\,\,{\rm for}\,\,\,\,x\ge
0\,,  \nonumber \\   U( x )&=&0\,\,\,\,\,\,\,\,{\rm
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\begin{eqnarray}
{\cal D}({\alpha
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\infty}   {\rm d}z\,{C_n}^2(z)\left[ {( {\alpha \kern 1ptD}  )^{5/
3}+d^{5/ 3}(z)}\right.  \nonumber\\
&&\left.
{-{\slantfrac{1}{2}}\,\left| {{\bbox \alpha} \kern 1ptD+{\bbox
d}(z)\,} \right|^{5/ 3} -{\textstyle \slantfrac{1}{2}}\left|
{\,{\bbox \alpha}  \kern 1ptD-{\bbox d}(z)\,} \right|^{5 / 3}}
\right],
\end{eqnarray}
where  ${C_n}^2(z)$  is the turbulence
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{}{{\Gamma\,\left[ {\lambda +m}  \right]\,\Gamma\,\left[ {\lambda
+p-m} \right]\cos \left[ {(p-2m)\varphi }  \right]} \over
{m!\,(p-m)!\,\left( {\Gamma\,\left[ \lambda  \right]}
\right)^2}},\end{eqnarray}
```

where - $\Gamma[x]$ is the gamma function. A particular Gegenbauer polynomial that is required is

$$C_2{}^{-5/6}[\cos(\varphi)] = \tfrac{5}{6}\left[1 - \tfrac{1}{3}\cos^2(\varphi)\right]. \tag{9}$$

For $\alpha D > d(z)$, the terms in the structure function can be expanded in Gegenbauer polynomials. The zeroth- and all odd-order terms cancel. When the summation index is changed by the substitution $p \to 2p$ the result is

$$\mathcal{D}(\alpha D) = 2(2.91)\,k_0{}^2 \int\limits_0^\infty \mathrm{d}z\, C_n{}^2(z) \left\{ d^{5/3}(z) - (\alpha D)^{5/3} \sum_{p=1}^{\infty} C_{2p}{}^{-5/6}\,[\cos(\varphi)] \left[\frac{d(z)}{\alpha D}\right]^{2p} \right\}. \tag{10}$$

It is this canceling of the first two terms of the power series that would cause numerical difficulties.

Define a distance moment as

$$d_m \equiv 2.91\,k_0{}^2 \int\limits_0^\infty \mathrm{d}z\, C_n{}^2(z)\, d^m(z) \tag{11}$$

and a phase variance as

$$\sigma_\varphi{}^2 = d_{5/3}. \tag{12}$$

Unlike the calculation for Strehl ratio for uncorrected turbulence and for corrected turbulence with tilt jitter, an exact analytical solution cannot be found for anisoplanatism. Fortunately, for adaptive-optics systems, the Strehl ratio should be fairly high by design, which requires the structure function to be small. This assumption allows one to retain only the first term of the Gegenbauer expansion to give

$$\mathcal{D}(\alpha D) = 2\sigma_\varphi{}^2 - 2x, \tag{13}$$

where

$$x = d_2\left[1 - \tfrac{1}{3}\cos^2(\varphi)\right]\ \tfrac{5}{6}(\alpha D)^{-1/3}. \tag{14}$$

We justify this single-term approximation below by showing that it produces a result close to the exact result.

. . .

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3}\sum\limits_{p=1}^\infty {{C_{2p}}^{- 5/ 6}\,\left[{\cos \left(
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 We justify this single-term approximation below by showing that it
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The Strehl ratio with the six term approximation is

$$\mathrm{SR} \approx \frac{\exp\left(-\sigma_\varphi{}^2\right)}{2\pi} \int \mathrm{d}\boldsymbol{\alpha}\, \mathrm{K}(\alpha) \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120}\right). \tag{15}$$

If just the first term in the last parenthetical expression is retained, the result is equivalent to the extended Maréchal approximation. It is shown below that the six-term approximation is best for aperture sizes normally encountered. The angle integral for the nth term, after use of the binomial theorem, is proportional to

$$\Phi(n) = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\varphi \left[1 - {}^1\!/_3 \cos^2(\varphi)\right]^n = \frac{1}{2\pi} \sum_{m=0}^{n} \begin{pmatrix} n \\ n-m \end{pmatrix} 3^{-m} \int_0^{2\pi} \mathrm{d}\varphi \cos^{2m}(\varphi), \tag{16}$$

$$\begin{pmatrix} n \\ n-m \end{pmatrix} = \frac{n!}{(n-m)!\, m!}. \tag{17}$$

Equation (4.641 # 4) in Gradshteyn and Ryzhik[8] is

$$\int_0^{\pi/2} \mathrm{d}\varphi \cos^{2m}(\varphi) = \frac{\pi(2m-1)!!}{2(2m)!!}, \tag{18}$$

where

$$(2m-1)!! = (2m-1)(2m-3)\ldots(3)(1), \tag{19}$$

$$(2m)!! = (2m)(2m-2)\ldots(4)(2). \tag{20}$$

With these relations, the angle integral is equal to

$$\Phi(n) = 1 - \sum_{m=1}^{n} \begin{pmatrix} n \\ n-m \end{pmatrix} 3^{-m} \frac{(2m-1)!!}{(2m)!!}. \tag{21}$$

The values of interest to us are $\Phi(0) = 1$, $\Phi(1) = 0.8333$, $\Phi(2) = 0.7083$, $\Phi(3) = 0.6134$, $\Phi(4) = 0.5404$, and $\Phi(5) = 0.4836$. The aperture integration for the nth term is proportional to

$$Y(n) = \int_0^1 \mathrm{d}\alpha\, \alpha^{1-n/3} K(\alpha). \tag{22}$$

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The
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\begin{eqnarray}{\rm   SR} \approx  {{\exp \left( {-\sigma
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\over {2\pi  }}\sum\limits_{m=0}^n {\left( \begin{array}{c} n \\
n-m\end{array}  \right)}\,3^{-m}\int\limits_{\,\,\, 0}^{\,\,\,\,\,\,\, 2\pi
} {\rm d\varphi }\, \cos ^{2m}\left( \varphi
\right),\end{eqnarray}
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\begin{array}{c} n \\ n-m \end{array} \right)={{n!} \over {\left(
{n-m} \right)!\,\,m!}}.\end{eqnarray}
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{2(2m)!!}},\end{eqnarray}
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\begin{eqnarray}(2m-1)!!&=&(2m-1)(2m-3)\ldots (3)(1), \\
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```

3. DISPLACEMENT ANISOPLANATISM

In the simplest case of displacement anisoplanatism, which was treated in Section 2, the displacement is constant along the propagation direction. The terms to use to find the Strehl ratio are

$$d(z) = d, \tag{23}$$

$$d_2 = 2.91\, k_0^2\, \mu_0\, d^2, \tag{24}$$

$$E = 6.88 \left(\frac{d}{D}\right)^2 \left(\frac{D}{r_o}\right)^{5/3}, \tag{25}$$

$$\sigma_\varphi^2 = 2.91\, k_0^2\, \mu_0\, d^{5/3} = 6.88 \left(\frac{d}{r_o}\right)^{5/3}. \tag{26}$$

The Strehl ratios are plotted in Figs. 2 and 3.

4. ANGULAR ANISOPLANATISM

When the propagation beam is offset by a constant angle from the direction along which turbulence is measured, the effect is called angular anisoplanatism.[4] ...

5. TIME DELAY

If there is a time delay between when turbulence is measured and when a correction is applied to the deformable mirror, there is a degradation in performance.[7] This effect is not often thought of as an anisoplanatic effect; however, it can be treated as such. ...

$$d(z) = v(z)\tau, \tag{27}$$

$$d_2 = 2.91\, k_0^2 \int_0^L \mathrm{d}z\, {C_n}^2(z)\, v^2(z)\, \tau^2 = (\tau/\tau_2)^2, \tag{28}$$

$$E = \frac{\tau^2}{\tau_2^2 D^{1/3}}, \tag{29}$$

$$\sigma_\varphi^2 = 2.91\, k_0^2 \int_0^L \mathrm{d}z\, {C_n}^2(z)\, v^{5/3}(z)\, \tau^{5/3} = \left(\tau/\tau_{5/3}\right)^{5/3}, \tag{30}$$

```
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\label{da}
In the simplest case of displacement  anisoplanatism, which was
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d_{\,2}&=&2.91\,k_0^2\,\mu _0\,d^2  ,     \\ E&=&6.88\,\left( {{d
\over D}} \right)^2\left(  {{D \over {r_o}}} \right)^{5/3}  ,
 \\ \sigma _\varphi ^2&=&2.91\,k_0^2\,\mu _0\,d^{5/3}=6.88\, \left(
{{d \over {r_o}}} \right)^{5/3}  .  \end{eqnarray}

The Strehl
ratios are plotted in Figs.~\ref{f5}  and ~\ref{f10}.

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d}z\,{C_n}^2(z)\,v^2(z)\,\tau ^2=\left(  {\tau / \tau _2} \right)^2
,   \\    E&=&{{\tau ^2} \over {\tau _2^2D^{1/ 3}}}  ,  \\  \sigma
_\varphi ^2&=&2.91\,k_0^2\int\limits_{\,\,\, 0}^{\,\,\,\,\,\, L}  {\rm
d}z\,{C_n}^2(z)\,v^{5/ 3}(z)\,\tau ^{5/ 3}=\left( {\tau / \tau _{5/
3}} \right)^{5/ 3}  ,   \end{eqnarray}
```

where the temporal moment is defined as

$$1/\tau_m^{5/3} = 2.91\, k_0^2 \int_0^L \mathrm{d}z\, {C_n}^2(z)\, v^m(z). \tag{31}$$

...

6. CHROMATIC ANISOPLANATISM

If the beacon beam that senses the turbulence has a wavelength different from that of the laser beam that is sent out, then the two beams will follow different paths through the atmosphere because of the dispersive properties of the atmosphere. ...

7. COMBINED DISPLACEMENT

If there are several anisoplanatic effects present, with each not decreasing the Strehl ratio much, it is a common practice to multiply the Strehl ratios for the individual effects to get a combined Strehl ratio. ...

$$\mathbf{d}_t(z) = \mathbf{d} + \boldsymbol{\theta} z + \mathbf{v}(z)\tau + \mathbf{d}_c(z), \tag{32}$$

where chromatic displacement is given in Eq. (50). The two terms necessary for calculating the Strehl ratio are ...

```
where the temporal moment is
defined as  \begin{eqnarray}  1/ \tau _m^{5/
3}=2.91\,k_0^2\int\limits_{\,\,\, 0}^{\,\,\,\,\,\, L}  {\rm
d}z\,{C_n}^2(z)\,v^m(z)  .   \end{eqnarray}
\ldots

\section{ CHROMATIC ANISOPLANATISM}
\label{ca}
If the beacon beam that senses the  turbulence has a wavelength
different from that of the laser beam that  is sent out, then the
two beams will follow different paths through the  atmosphere
because of the dispersive properties of the atmosphere. \ldots

\section{ COMBINED DISPLACEMENT}
\label{cd}
If there are several anisoplanatic  effects present, with each not
decreasing the Strehl ratio much, it is a  common practice to
multiply the Strehl ratios for the individual effects  to get a
combined Strehl ratio. \ldots
\begin{eqnarray}
{\rm \pmb{d}}_t(z)={\rm  \pmb{d}}+{ \rm \bbox{
\theta}} \kern 1ptz+{\rm \pmb{v}}(z)\tau +{\rm  \pmb{d}}_c(z)  ,
\end{eqnarray} where chromatic displacement is given in Eq. (50).
The two terms  necessary for calculating the Strehl ratio are
\ldots
```

8. SUMMARY

An approximate expression for the Strehl ratio that is easily evaluated for any turbulence distribution was derived. It applies for various anisoplanatic effects. This expression was shown to give much better agreement with the exact answer than the extended Marechal approximation. The zenith dependence is included in the formula. This approximation was applied to parallel path displacements, angular offsets, time-delay induced offsets, and offsets owing to refractive effects that vary with wavelength. Examples for each type of anisoplanatism at various zenith angles were evaluated.

The Strehl ratio in the presence of several effects was examined. It was shown that, depending on the direction of the relative displacements, one can get a cancellation or an enhancement of the effect of the displacements. Therefore it is possible for there to be little reduction in the Strehl ratio if there is little net path displacement. If the displacements are in the same direction, the Strehl ratio is less than the product of the Strehl ratios of the individual terms.

ACKNOWLEDGMENTS

This research was sponsored by the Strategic Defense Initiative Organization through the U.S. Department of the Air Force.

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\end{references}

FIGURES

Fig. 1. Comparison of the Maréchal and the two- to six-term approximations with the exact value of the Strell ratio, for an anisoplanatic displacement, for D/r_0 equal to 1.

Fig. 2. Comparison of the Maréchal and the two- to six-term approximations with the exact value of the Strell ratio, for an anisoplanatic displacement, for D/r_0 equal to 5.

Fig. 3. Comparison of the Maréchal and the two- to six-term approximations with the exact value of the Strell ratio, for an anisoplanatic displacement, for D/r_0 equal to 10.

Fig. 4. Strehl ratio for angular anisoplanatic error at zenith, for various turbulence models, versus separation angle for a 0.6-m system. Upper-altitude turbulence has a strong effect on the Strehl ratio.

Fig. 5. Strehl ratio for angular anisoplanatism at 30° for a 0.6-m system.

Fig. 6. Strehl ratio versus time delay at zenith for a 0.6-m system.

Fig. 7. Strehl ratio versus time delay for a 0.6-m system at 30° zenith angle. Strehl ratio at 30° for a 0.6-m system.

Fig. 8. Difference ($\times 10^6$) in refractive index between $0.5\,\mu$m and other wavelengths.

```
\begin{figure}
\caption{ Comparison of the Mar\'{e}chal and the two- to six-term
approximations  with the exact value of the Strell ratio, for an
anisoplanatic displacement, for $D/r_0$  equal to 1.}\label{f1}
\end{figure}

\begin{figure}
\caption{ Comparison of the Mar\'{e}chal and the two- to six-term
approximations  with the exact value of the Strell ratio, for an
anisoplanatic displacement, for $D/r_0$  equal to 5. } \label{f5}
\end{figure}
\begin{figure}
\caption{ Comparison of the Mar\'{e}chal and the two- to six-term
approximations  with the exact value of the Strell ratio, for an
anisoplanatic displacement, for $D/r_0$  equal to 10. } \label{f10}
\end{figure}
\begin{figure}
\caption{Strehl ratio for angular anisoplanatic error at zenith,
for  various turbulence models, versus separation angle for a 0.6-m
system.   Upper-altitude turbulence has a strong effect on the
Strehl ratio.}
\label{faaz}
\end{figure}
\begin{figure}
\caption{ Strehl ratio for angular anisoplanatism at $30^{\circ}$
for a 0.6-m system.}
\label{faa30}
\end{figure}
\begin{figure}
\caption{ Strehl ratio versus time delay at zenith for a 0.6-m
system.}
\label{ftdz}
\end{figure}
\begin{figure}
\caption{ Strehl ratio versus time delay for a 0.6-m system at
$30^{\circ}$ zenith angle.     Strehl ratio  at $30^{\circ}$ for a
0.6-m system. }
\label{ftd30}
\end{figure}
\begin{figure}
\caption{ Difference ($\times 10^6$) in refractive index between
$0.5 \, \mu \rm m$ and other wavelengths.}\label{fri}
\end{figure}
```

TABLES

Table 1. Values of T_2 and $T_{5/3}$ to Solve for the Chromatic Displacement for Various Turbulence Models for a Wavelength of 0.5 μm

Model	$T_2{}^a$	$T_{5/3}{}^b$
SLC-Day	2.71×10^{-6}	2.00×10^{-7}
HV-21	6.16×10^{-6}	3.60×10^{-7}
HV-54	3.40×10^{-5}	1.87×10^{-6}
HV-72	5.95×10^{-5}	3.25×10^{-6}

[a] The units of T_2 are $m^{1/3}$.

[b] $T_{5/3}$ is dimensionless.

```
\begin{table}
\caption{Values of $T_2$ and  $T_{5/3}$  to Solve for the Chromatic
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$\mu \rm m$}
\begin{tabular}{lcc}
Model&$T_2$\tablenote{The units of $T_2$ are $m^{1/3}$.}&
$T_{5/3}$\tablenote{$T_{5/3}$  is dimensionless.} \\ \tableline
SLC-Day&$2.71 \, \times \, 10^{-6}$&$2.00 \, \times \, 10^{-7}$\\
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\end{tabular}
\end{table}

\end{document}
```

Generation, propagation, and amplification of dark solitons

W. Zhao and E. Bourkoff

Department of Electrical and Computer Engineering, The University of South Carolina, Columbia, South Carolina, 29208

Abstract

The technique for generating dark solitons with constant background using guided-wave Mach–Zehnder interferometers is further examined. Under optimal conditions, a reduction of 30% in both the input optical power and the driving voltage can be achieved, as compared with the case of complete modulation. Dark solitons are also found to experience compression through amplification. When the gain coefficient is small, adiabatic amplification is possible. Raman amplification can be used as the gain mechanism for adiabatic amplification, in addition to being used for loss-compensation. The frequency and time shifts caused by intrapulse stimulated Raman scattering are both found to be a factor of 2 smaller than those for bright solitons. Finally, the propagation properties of even dark pulses are described quantitatively.

1. INTRODUCTION

Nonlinear optical pulses can propagate in dispersive fibers in the form of bright and dark solitons under certain conditions, as first described by Zakharov and Shabat in 1972[1] and in 1973,[2] respectively. They are stationary solutions of the initial boundary value problem of the nonlinear Schrödinger equation (NLSE).[3]

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\documentstyle[osa,manuscript]{revtex}  % DON'T CHANGE
%
%
\newcommand{\MF}{{\large{\manual META}\-{\manual FONT}}}
\newcommand{\manual}{rm}        % Substitute rm (Roman) font.
\newcommand\bs{\char '134 }     % add backslash char to \tt font
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%
\begin{document}                % INITIALIZE - DONT CHANGE
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%
\title{Generation, propagation, and amplification of dark solitons}
%
\author{W. Zhao and E. Bourkoff}
%
\address{Department of Electrical and Computer Engineering,
The University of South Carolina,
Columbia, South Carolina, 29208}
%
\maketitle
\begin{abstract}                % DON'T CHANGE THIS LINE
The technique for generating dark solitons with constant background
using guided-wave Mach--Zehnder interferometers is further examined.
Under optimal conditions, a reduction of 30\% in both the input optical power
and the driving voltage can be achieved, as compared with the case of
complete modulation.  Dark solitons are also found to experience compression
through amplification.  When the gain coefficient is small, adiabatic
amplification is possible.  Raman amplification can be used as the gain
mechanism for adiabatic amplification, in addition to being used for
loss-compensation.  The frequency and time shifts caused by intrapulse
stimulated Raman scattering are both found to be a factor of 2 smaller
than those for bright solitons.  Finally, the propagation properties of
even dark pulses are described quantitatively.
\end{abstract}

\section{ INTRODUCTION}
\label{INT}
Nonlinear optical pulses can propagate in dispersive fibers in the form of
bright and dark solitons under certain conditions,
as first described by Zakharov and Shabat in 1972\cite{ZA}
and in 1973,\cite{ZB}
respectively.
They are stationary solutions of the initial boundary value problem of the
nonlinear Schr{$\rm\ddot o$}dinger equation (NLSE).\cite{SA} \ldots
```

In the anomalous dispersion regime of the fiber, under the boundary condition $u(z, t = \pm\infty) = 0$, there exists a class of particle-like, stationary solutions called bright solitons.[4] In the normal dispersion region, under the boundary condition $|u(z, t = \pm\infty)|$ =constant, one can obtain another class of stationary solutions, which are called dark solitons, since a dip occurs at the center of the pulse.[5] ...

In the following discussions, we adopt the normalization convention used in Agrawal's book.[6] We normalize the field amplitude A (optical power $P_0 = A^2$) into u by

$$u = \left(\frac{2\pi n_2 {\tau_0}^2}{\lambda A_{\text{eff}} |\beta_2|} \right)^{1/2} A,$$

where A_{eff} is the effective area of the propagating mode, $n_2 = 3.2 \times 10^{-16}\text{cm}^2/\text{W}$ is the nonlinear optical Kerr coefficient of the silica fiber, and β_2 is a parameter describing the group velocity dispersion of fiber, ...

2. GENERATION OF DARK SOLITONS

In our earlier work[7,8] we discussed the possibility of using an integrated Mach–Zehnder interferometer (MZI) to generate dark solitons with constant background. ...

... Therefore the pulse after the MZI, when properly biased, can have the form

$$u(0, t) = a \sin[\delta\pi/2 \tanh(t)], \tag{1}$$

3. PROPAGATION AND AMPLIFICATION

As discussed in Section 2, when smaller values of δ are used, pulses of better characteristics are obtained. This can be seen in Fig. 1(d), where $a = 1.33$ and a pure fundamental dark soliton is generated. ....

```
In the anomalous dispersion regime of the fiber, under the boundary
condition $  u( z, t = \pm \infty ) = 0 $, there exists
a class of particle-like, stationary solutions
called bright solitons.\cite{HA}
In the normal dispersion region, under the boundary condition
$ | u( z, t = \pm \infty ) | = $constant,
one can obtain another class of stationary solutions,
which are called dark solitons, since a dip occurs at the
center of the pulse.\cite{HB} \ldots

In the following discussions, we adopt the normalization convention
used in Agrawal's book.\cite{AB}
We normalize the field amplitude $A$ (optical power $P_0 = A^2 $)
into $u$ by
\begin{eqnarray*}
u = \left( { 2 \pi n_2 {\tau_0}^2 }\over
{ \lambda A_{\rm eff} | \beta_2 | } \right)^{1/2} A ,
\end{eqnarray*}
where $A_{\rm eff }$ is the effective area of the propagating
mode, $n_2 = 3.2\times 10^{-16}$cm$^2 /$W is the nonlinear optical
Kerr coefficient of the silica
fiber, and $ \beta_{2} $ is a parameter describing
the group velocity dispersion of
fiber, \ldots

\section{GENERATION OF DARK SOLITONS}
\label{GDS}
In our earlier work\cite{ZBD,ZBE} we discussed the possibility
of using an integrated Mach--Zehnder interferometer
(MZI) to generate dark solitons with constant background. \ldots

 \ldots Therefore the pulse after the MZI,
when properly biased, can have the form
\begin{eqnarray}
u (0,t)  =  a\, {\rm sin} [ \delta \pi /2\, {\rm tanh} (t) ],
\label{E1}
\end{eqnarray}

\section{PROPAGATION AND AMPLIFICATION}
\label{PAA}
As discussed in Section \ref{GDS}, when smaller values of
$ \delta $ are used, pulses of better
characteristics are obtained.  This can be seen in Fig. 1(d),  where
$ a  =  1.33 $ and a pure fundamental dark soliton is
generated. \ldots .
```

We first examine the solution of a modified NLSE with a constant gain:

$$iu_z - 1/2u_{tt} + |u|^2u = i\Gamma u, \tag{2}$$

where Γ is assumed to be a constant, appropriate for the Raman amplification under strong pumping without depletion. The solution of a similar equation to Eq. (2), but ...

$$t' = te^{\Gamma z}, \tag{3a}$$

$$z' = \frac{e^{2\Gamma z} - 1}{2\Gamma}, \tag{3b}$$

$$u = ve^{\Gamma z}. \tag{3c}$$

Under this transformation, the NLSE has the new form

$$iv_{z'} - \tfrac{1}{2}v_{t't'} - |v|^2v = -\frac{\Gamma t'}{2\Gamma z' + 1}v_{t'}. \tag{4}$$

The solution of Eq. (2) when $\Gamma = 0$ is well known and has the form $\exp[i\sigma(z,t)]\kappa \tanh \kappa t$, where κ is the form factor and the phase variable satisfies $\partial\sigma/\partial z = \kappa^2$.[1] Therefore, when the right-hand-side of Eq.(4) is zero, an exact solution for $v(z', t)$ can be obtained from Eq. (4). On the other hand, when $z \to \infty$ and hence $z' \to \infty$ or $\Gamma \to 0$, the right-hand side of Eq. (4) becomes infinitely small. Under these conditions, we can treat the right-hand side of Eq. (4) as a perturbation to the NLSE. ...

$$u(z,t) = \exp\left(i\frac{e^{2\Gamma z} - 1}{2\Gamma}\right) e^{\Gamma z} \tanh(te^{\Gamma z}), \tag{5}$$

$$\Gamma = g(e^{-2\Gamma_p z} + e^{-2\Gamma_p (L-z)}) - \Gamma_s, \tag{6}$$

$$g = \frac{\Gamma_p(\Gamma_s + \beta)L}{\sinh(\Gamma_p L)} e^{\Gamma_p L}, \tag{7}$$

$$\kappa(z) = \kappa_0 \exp(\beta z). \tag{8}$$

4. EFFECTS OF INTRAPULSE STIMULATED RAMAN SCATTERING

The properties of dark solitons considered thus far are based on the simplified propagation equation (2). ...

```
We first examine the solution of a modified NLSE with a constant gain:
\begin{eqnarray}
i u_{z} - {1/2} u_{tt} + |u|^2 u  =  i \Gamma u,
\label{E2}
\end{eqnarray}
where $\Gamma $ is assumed to be a constant,
appropriate for the Raman amplification
under strong pumping without depletion.  The
solution of a similar equation to Eq. (\ref{E2}),
but \ldots
\begin{mathletters}
\begin{eqnarray}
t’  &=&  t e^{ \Gamma z }, \label{E4}  \\
z’  &=&  { e^{2 \Gamma z } - 1 \over  2 \Gamma },  \label{E5}  \\
u   &=&  v e^{ \Gamma z } .                          \label{E6}
\end{eqnarray}
\end{mathletters}
Under this transformation,  the NLSE has the new form
\begin{eqnarray}
i v_{z’} -\slantfrac{1}{2} v_{ t’ t’ } - |v|^2 v  &=&
- { \Gamma t’ \over 2 \Gamma z’ + 1} v_{t’}. \label{E7}
\end{eqnarray}
The solution of Eq. (\ref{E2}) when $\Gamma $ = 0 is well known and
has the form ${\rm exp} [i \sigma (z,t) ] \kappa \tanh \kappa t $,
where $\kappa $ is the form factor and the phase variable satisfies
$ \partial \sigma / \partial z  =  \kappa^2 $.\cite{ZA}
Therefore, when the right-hand-side of
Eq.(\ref{E7}) is zero, an exact solution for $v(z’,t)$ can be
obtained from Eq. (\ref{E7}).
On the other hand, when $z \rightarrow \infty $ and hence
$z’ \rightarrow \infty $ or $ \Gamma \rightarrow 0$,  the
right-hand side of Eq. (\ref{E7}) becomes infinitely small.
Under these conditions, we can treat the right-hand
side of Eq. (\ref{E7}) as a perturbation to the NLSE.
\ldots
\begin{eqnarray}
u(z,t)&=&{\rm exp}\left( i{e^{2\Gamma z}-1 \over 2\Gamma}\right)
e^{\Gamma z} \, {\rm tanh} (te^{\Gamma z}),
\label{E8}                               \\
\Gamma&=&g(e^{-2\Gamma_pz} + e^{-2\Gamma_p(L-z)}) - \Gamma_s,
\label{E9} \\
g&=&{\Gamma_p(\Gamma_s + \beta)L \over {\rm sinh}(\Gamma_pL)}
e^{\Gamma_pL} , \label{E10} \\
\kappa(z) &=& \kappa_0 \, {\rm exp}(\beta z). \label{E11}
\end{eqnarray}

\section{EFFECTS OF INTRAPULSE STIMULATED RAMAN SCATTERING}
\label{EIS}
The properties of dark solitons considered thus far are
based on the simplified propagation equation (\ref{E2}).
\ldots
```

…The energies of these separating solitons are distributed in such way to ensure conservation of momentum. …

$$iu_z - 1/2u_{tt} + |u|^2u = \tau_d \frac{\partial |u|^2}{\partial t} u, \tag{9}$$

5. EVEN DARK PULSES

Even dark pulses,[9,10] which are symmetric functions of time centered around the pulse, can be simply generated by driving the MZI with a short electric pulse. …

If we define the amplitudes of the secondary soliton pairs as

$$\kappa_n = \kappa_0 - \Delta_n, \tag{10}$$

then the nth order secondary pulse shape (n = 1, 2, 3, …) has the form

$$u_n(z,t) = \kappa_0 \frac{(\lambda_n - i\nu_n)^2 - \nu_n \exp[2\nu_n(t - t_{n0} - \lambda_n z)]}{1 + \nu_n \exp[2\nu_n(t - t_{n0} - \lambda_n z)]} e^{iz}, \tag{11}$$

…

6. CONCLUSIONS

We have discussed the possibility of using the waveguide Mach–Zehnder interferometer to generate a variety of dark solitons under constant background. Under optimal operation, 30% less input power and driving voltage are required than for complete modulation. The generated solitons can have good pulse quality and stimulated Raman scattering process can be utilized to compensate for fiber loss and even to amplify and compress the dark solitons. …

ACKNOWLEDGMENTS

The authors thank the reviewers for their constructive comments. This research was supported by National Science Foundation grant ECS-91960-64.

```
\ldots The energies of these
separating solitons are distributed in such way to ensure conservation of
momentum.                                                    \ldots

\begin{eqnarray}
iu_z - {1/2}u_{tt}+|u|^2u &=& \tau_d{\partial |u|^2 \over \partial t}u,
\label{E12}
\end{eqnarray}
\ldots

\section{EVEN  DARK PULSES}
\label{EDP}
Even dark pulses,\cite{KA,WA} which are symmetric functions of time
centered around the pulse, can be simply generated by driving
the MZI with a short electric pulse. \ldots

If we define the amplitudes of the secondary soliton pairs as
\begin{eqnarray}
\kappa_n  =  \kappa_0 - \Delta_{n} ,  \label{E16}
\end{eqnarray}
then the $n$th order secondary pulse shape (n = 1, 2, 3, \ldots )
has the form
\begin{eqnarray}
u_n (z,t)  =  \kappa_{0}{(\lambda_n - i \nu_n )^2 - \nu_n
\,{\rm exp} [ 2 \nu_n (t-t_{n0} - \lambda_{n} z)] \over  1 +
\nu_n\, {\rm exp} [ 2 \nu_n (t-t_{n0} - \lambda_n z)]} e^{iz},
\label{E17}
\end{eqnarray}
\ldots

\section{CONCLUSIONS}
We have discussed the possibility of using the waveguide Mach--Zehnder
interferometer to generate a variety of dark solitons under constant
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Fig. 1. The dark solitons generated by the waveguide Mach-Zehnder interferometer. The amplitude of the input cw light is chosen to be $a = \pi/2$ for (a)-(c). The parameter δ is (a) 0.8, (b) 0.5, and (c) 0.2. Part (d) is the case of optimal operation when $a = 1.33$, and $\delta = 0.7$. In all cases, the output pulse shapes are plotted as solid curves while the dashed curves are input pulse shapes. The pulses shown here are at a propagation distance of $z = 4$.

Fig. 2. Dark solitons under constant gain. Pulse shapes (solid) when Γ=0.05 (a) and 1(b), after certain propagation distance, Γz=1.6, as compared to input pulse shapes (dashed). (c): The pulse duration, relative to its input, as a function of Γz at various Γ. The solid curve is the adiabatic approximation obtained by perturbation method. Three values of Γ are used: $\Gamma = 0.05$ (dotted); 0.2 (dash-dotted); and 1 (dashed). Negative Γz depicts the case of loss.

Fig. 3. The pulse shapes of amplified dark solitons. (a) $\delta = 0.5$, $\beta = 2ln1.05$, $\Gamma_p L = 2$, after 8 amplifying cycles (solid); (b) $\delta = 0.5$, $\beta = 2ln1.02$, $\Gamma_p L = 2$, after 16 amplifying cycles (solid); (c) $\delta = 0.5$, $\beta = 2ln1.02$, $\Gamma_p L = 0.5$, after 16 amplifying cycles (solid); (d) The input pulse is the same as in Fig. 1(c), $\beta = 2ln1.05$, after 8 amplification periods (solid). The input pulse shapes are plotted as dashed curves.

Fig. 4. (a) The shape of a fundamental dark soliton after a propagation distance of 40 (solid). The normalized time delay $\tau_d = 0.01$. The dashed curve is the input pulse shape. (b) The trace of the soliton (solid) as a function of propagation distance for the situation described by (a). The dotted curve represents the case for a fundamental bright soliton under similar conditions.

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Table 2. Amplitudes of Secondary Even Dark Pulses

	Input Pulse Shape				
Δ_n Values	$\kappa_0 \lvert \tanh t \rvert$	$\kappa_0[1-\exp(-t^2/\tau_g{}^2)]^{1/2}$	$\kappa_0[1-\mathrm{sech}(t/\tau_s)]$	Avg.	Range
Δ_1	0.34	0.30	0.21	0.28	±25%
Δ_2	1.56	1.41	1.26	1.41	±11%
Δ_3	2.47	2.26	2.28	2.34	±6%
Δ_4	3.52	3.25	3.31	3.36	±6%
Δ_5	4.45	4.26	4.42	4.38	±6%
Δ_6	5.52	5.35	5.50	5.50	±5%

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Designing digital optical computing systems: power distribution and cross talk

Jonathan P. Pratt and Vincent P. Heuring

When this work was performed, both the authors were with the Boulder Optoelectronic Computing Systems Center and Department of Electrical and Computer Engineering, University of Colorado, Campus Box 425, Boulder, Colorado 80309-0425. They are now with the Department of Radiology, University of Colorado Health Sciences Center, Box A034, 4200 East Ninth Avenue, Denver, Colorado 80262.

Abstract

Complex optical computer designs must implicitly or explicitly allow for power budgeting, to compensate for cross talk and loss in both devices and interconnections. We develop algorithms for calculating the system cross talk and power loss in optical systems, using a graph-theoretic model. Devices are modeled as directed graphs with nodes representing inputs and outputs, and edges are weighted with the power relationships between nodes. Systems are modeled by interconnecting the individual device graphs in a manner that reflects the connectivity of the system. A system's power budget is efficiently computed by a depth-first search of its graph. The algorithms have been incorporated into an optical computer-aided design system that is now being used to design a bit-serial optical computer containing hundreds of components.

Key words: Optical computing, optical systems, optical communications, power loss, cross talk, graphs.

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\newcommand{\MF}{{\large{\manual META}\-{\manual FONT}}}
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\author{Jonathan P. Pratt and      Vincent P. Heuring}

\address{
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\maketitle

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1. Introduction

We describe a technique that facilitates the design of digital optical computers and other complex optical circuitry, such as optical communications systems. Although there has been some discussion in the literature of power budgeting in optical systems,[1,2] the treatment has been limited to relatively uncomplicated applications, ...

2. Power Loss and Cross Talk in the System

A. Introduction

Appropriate signal levels must be maintained in any digital optical system that uses signal level thresholds to encode transmitted information. Usually a high-level signal represents a logic 1 and a low-level signal represents a logic 0. In these systems the device characteristics of importance are power loss and cross talk.

...

B. Power Levels and Correct Device Operation

Here we discuss the type of power information desired from a system model. Since the objective is to find weak points in the system power flow, only power extremes are considered. Power extremes are the cross talk and signal levels obtained when the worst possible combinations of device states and input power levels are assumed.

... The weakest 1 arriving at the detection point under all conditions from all possible paths to the point is defined as $P_{1\min}$, and similarly, the strongest 0 is defined as $P_{0\max}$. Proper device operation can be ensured if the following relations are met:

$$P_{0\max} < P_{S2} < P_D < P_{S1} < P_{1\min}. \tag{1}$$

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It is also desirable to have information about $P_{\max}$, the maximum power level that can occur at the inputs to a given device. A power detector may provide erroneous results when the power of a logic 1 arriving at a detection point is too large; that is, when $P_{\max}$ exceeds P_D by some large amount. A second and more important reason for computing $P_{\max}$ is that it makes the major contribution to cross talk, as discussed below. Knowledge of the power triple $P_{0\max}, P_{1\min}$, and $P_{\max}$ at each device in a system permits the tracking of power levels throughout the entire system.

C. Modeling the Device

Here we discuss the means for calculating the power triples $P_{0\max}$, $P_{1\min}$, and $P_{\max}$ at the outputs of a given device, given the values of the triples at each of its inputs. ...

...The power triple for the jth output of a device is computed from the input triples and the coupling terms as follows:

$$P_{1\min}(\mathrm{out})_j = \min_{s\,\in\,\mathrm{states}} \{ \min_{\mathrm{inputs}\,i} [P_{1\min}(\mathrm{in})_i - L_{ij}(s)]\}, L_{ij}(s) \in \mathrm{loss}, \tag{2}$$

$$P_{0\max}(\mathrm{out})_j = \max_{s\,\in\,\mathrm{states}} \sum_{\mathrm{inputs}\,i} \begin{cases} P_{\max}(\mathrm{in})_i - L_{ij}(s)\ ,\ L_{ij}(s) \in \text{cross talk}, \\ P_{0\max}(\mathrm{in})_i - L_{ij}(s)\ ,\ L_{ij}(s) \in \mathrm{loss}, \end{cases} \tag{3}$$

$$P_{\max}(\mathrm{out})_j = \max_{s\,\in\,\mathrm{states}} \sum_{\mathrm{inputs}\,i} P_{\max}(\mathrm{in})_i - L_{ij}(s). \tag{4}$$

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in})_i - L_{ij}(s)\; ,\; L_{ij}(s)\in {\rm loss},   \end{array}
\right. \\ P_{\rm max}({\rm out})_j &=& \begin{array}[t]{c}{\rm
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i}  P_{\rm max}{\rm (in)}_i - L_{ij}(s) . \label{pmax}
\end{eqnarray}
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Equation (2) states that the power of the minimum 1 emerging from the jth output of the device will be the minimum over all possible states of the minimum over all possible inputs having loss terms of the minimum 1's arriving at those inputs minus the loss terms. Equation (3) states that the power of the maximum 0 emerging from the jth output of the device will be the maximum over all possible states of the sum of the inputs ...

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In this section we extend the applicability of the device graph model to complete systems. ...

3. Discussion

The technique described above is indispensable in designing complex optical systems whose components have significant nonidealities. It has been incorporated into a digital optical computer-assisted design system, HATCH,[10] where it has proven invaluable in the design of optical counters and an optical delay line memory system. It is now being used in designing a bit-serial optical computer now under construction in our laboratories. ...

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Table 3. Minimum Signal Powers

Vertex	$P_{1\min}$ (dBm)
1	0
2	-3
3	-5
4	-5
5	-8
6	-11
7	-8

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